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Application of Multiple Linear Regression Method in Steel Column Design under Combined Loading

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ABSTRACT

The article explores using multiple linear regression (MLR) approach to predict the cross-section of beam-column steel members. These members are complex in design, as they must withstand bending and axial forces. The design process involves multiple safety checks according to design standards. The first step in the design procedure is to calculate and verify the axial capacity of the member. Following that, the flexural capacity is calculated. Finally, the combined load capacity is examined, often involving a trial-and-error process. If the initially selected section cannot withstand the applied forces, a new section must be chosen, and the calculations must be repeated. This study employs a multiple linear regression approach to investigate the relationship between member properties (such as length and cross-section) and load design parameters (axial and bending forces). The primary objective is to predict the beam-column steel member's elastic section modulus (S_x). The regression equation derived from the study is as follows: $S_x = 18.000L + 6.839P_r + 47.193M_r - 129.093$, with an R-square value of 0.985. The histogram of the equation exhibits symmetry and a standard distribution, while the typical probability plot demonstrates a relatively linear trend. Two examples are presented to evaluate the efficiency of the regression equation. The first example confirms the efficiency of the equation by determining the cross-sectional size by employing the regression equation and using it to verify the safety according to EIT design standards. The second example is the application of the equation for estimating the size of other types of steel members. This study suggests that the regression equation can effectively predict the size of beam-column steel members, providing reliable results for practical design applications.

1. Introduction

Continuous development has been observed in statistical regression theory [1-7]. The primary objective of regression theory is to examine the correlation between dependent and independent variables. When a regression model incorporates multiple independent variables, it is called a multivariate regression model. The multiple linear regression (MLR) analysis has found application across diverse fields, such as sciences, management, engineering, and education.

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For instance, Gulden and Nese [8] utilized a multilinear regression equation to predict student's KPSS scores at Sakarya University Education Faculty based on five significant independent variables. Febrianti and Viany [9], and Stephen *et al.,* [10] developed a multiple linear regression model to forecast students' performance. In economics, Turóczy and Liviu [11] formulated a profit size model in the ceramic industry using variables like self-financing capacity, return on equity, degree of technical endowment, investment per person employed, and personnel cost per employee. Liang [12] and Zhou *et al.,* [13] also studied China's economic growth through a multiple linear regression model. Considerable studies have applied linear regression methods to estimate stock market prices in financial [14-17]. In housing markets, Kang and Zhao [18] predicted housing prices using an improved multiple regression model incorporating fuzzy mathematics. In engineering, Lilian *et al.,* [19] employed a multiple regression approach to estimate traffic flow in Hong Kong utilizing three types of traffic datasets. In a different study, Chayalakshmi *et al.,* [20] investigated boiler efficiency, considering significant boiler losses caused by dry flue gas, hydrogen fuel, and fuel moisture content using multiple regression analyses. Moreover, Jaisumroum and Teeravaraprug [21] presented a multiple linear regression model to forecast Thailand's electricity consumption and compared it with an artificial neural network model. In the medical field, Edelman *et al.,* [22] applied a linear regression model to predict total procedure time (TPT) based on data from surgeries performed in six academic hospitals in The Netherlands.

In line with these varied applications of regression, this research introduces a linear regression technique to predict the steel column section under combined loading (compressive axial force and bending moment). Numerous studies have presented various approaches and methods in the complicated steel column design under combined force [23-26]. However, in some ways, it is still necessary to use computer programs to help make work easier. Therefore, this study presents another design approach by applying multiple linear regression theory to optimize the member size. The study compounds a database of 1,128 beam-column steel design cases that were transformed into a new equation of steel design.

2. Methodology

2.1 Problem and Objective of Study

In general, column members are primarily designed to handle axial loads. However, there are columns in some types of structures that may have to be designed to support the bending moment, such as columns in structures that must withstand lateral loads or earthquakes. The column under combined loading (compressive axial force and bending moment) names a beam-column member. A beam-column steel member's calculation and design methods are intricate, primarily due to the combined load behaviour between axial and flexural forces shown in Figure 1.

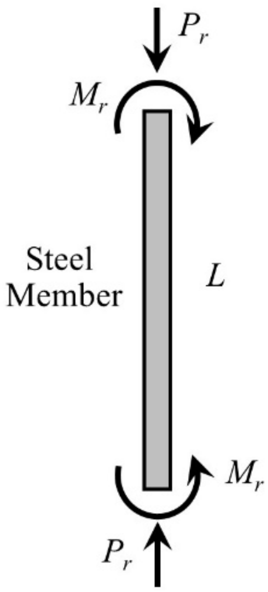

Fig. 1. Steel column member under combined loading

For the design of steel structures in general, an international standard will be used as a design guideline, AISC 360-16 [27]. In Thailand, the Engineering Institute of Thailand will give a control standard, namely EIT 011038 [28], referenced from AISC 360-16. For designing the beam-column member, the designer will calculate the ultimate strength of the member to be sufficient for the force received. The ultimate strength of the beam-column steel member can be determined by the axial and flexural strength relationship, as illustrated in Figure 2. Where P_r represents the required axial strength, P_c signifies the available axial strength of the cross-section, M_r represents the required flexural strength, and M_c signifies the available flexural strength of the cross-section.

Based on Figure 2, it is evident that an increase in axial capacity leads to a decrease in flexural capacity. Similarly, as the bending force increases, the allowable axial force diminishes. The available axial strength of the cross-section (P_c) is determined by the equation $P_c \leq \phi P_n$, where ϕ represents the resistance factor for compression (equal to 0.9), and P_n denotes the nominal compressive axial strength based on the chosen cross-section.

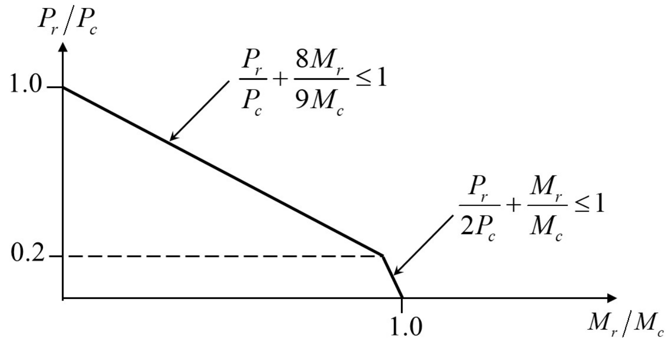

Fig. 2. Relationship between compressive strength and flexural strength

Figure 3 illustrates the nominal axial strength (P_n) of the steel cross-section, incorporating properties such as A_g (cross-sectional area), r (radius of gyration), F_y (yield strength of the steel), E

(elastic modulus of the steel), and L_c (length of the member). It can be seen that the axial strength of the member diminishes with an increase in the slenderness ratio (L_c/r). In simpler terms, elongating a member's length will result in a reduced permissible compressive force. A member with a high slenderness ratio tends to buckle when subjected to a compressive force. However, even with a lower slenderness ratio, a buckle can occur when the member is subjected to combined forces.

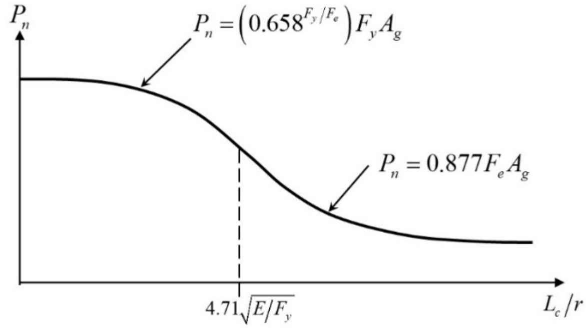

Fig. 3. The nominal axial strength of the compression member

Furthermore, based on the Thailand design standard [28], the available flexural strength is obtained from $M_c \leq \phi M_n$, where ϕ represents the resistance factor for flexure (equal to 0.9), and M_n denotes the nominal bending strength of the selected cross-section. The nominal flexural strength of the steel cross-section according to the conditions outlined in Figure 4, where Z_x (plastic section modulus), S_x (elastic section modulus), I_y (moment of inertia about the minor axis), J (polar moment of inertia), and C_w (warping constant) are the properties of the cross-section, F_y (yield strength of steel), E (elastic modulus) of steel), and G (shear modulus of steel) are the steel properties, L_b denotes unbraced length of member, L_p and L_r represent the limiting laterally unbraced length.

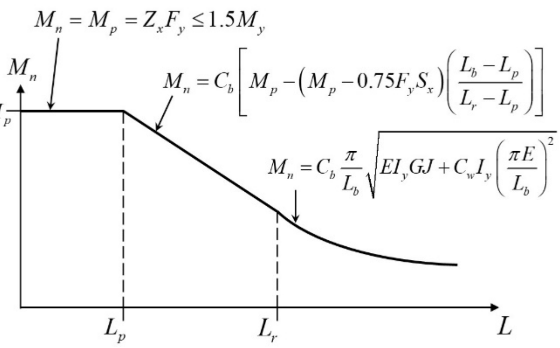

Fig. 4. The nominal bending strength of the flexural member

According to Figure 4, the plastic section modulus (Z_x) exists as the primary variable defining the allowable maximum bending moment of the cross-section. Also, elongating the member's length will decrease the permissible bending strength due to the influence of lateral torsion buckling (LTB) effects. Therefore, the strength of the beam-column member is influenced by both the *P-Δ* (due to lateral deformation) and lateral torsion buckling. These effects impact the member's axial and bending strength depending on the member's unbraced length and type of cross-section.

From the previous, it is readily apparent that many specifications of the ultimate strength requirements result from the load behaviour of members. The general steel beam-column design process typically involves the following steps. First, the designer selects a cross-section and assesses the axial strength ratio (P_r/P_c) to determine the appropriate design conditions. Next, the nominal bending strength of the chosen cross-section is calculated using the equation provided in Figure 4. Finally, the combined axial and flexural strengths are evaluated based on the conditions outlined in Figure 2. It is worth mentioning that the design of steel beam-column members often entails a trial-and-error approach. The designer selects an initial trial cross-section and examines its compliance with the equations in Figures 2 to 4. This iterative process helps ensure that a suitable cross-section is chosen, minimizing the risk of selecting an insufficient one. However, it may take a long time if the designer chooses a new cross-section for 2-3 rounds without a computer program to help calculate. The multiple linear regression method can simplify the calculation process and reduce the probability of selecting an insufficient cross-section. This approach facilitates the optimization of the cross-sectional properties of steel members subjected to combined forces, offering a more efficient and rapid design process.

2.2 Multiple Linear Regression Theorem

The multiple linear regression equation can be represented as:

$$Y = B_0 + B_1X_1 + B_2X_2 + ... + B_nX_n \tag{1}$$

In Eq. (1), Y represents the dependent variable. In this study, the variable that needs to be estimated or predicted is the section of the element. According to the steel table data, the cross-sectional area (A_g) of steel is shown, and same-size steel with different thicknesses will have a very close cross-sectional area. Since the value obtained from the regression equations is approximate, selecting a section from the steel table is difficult. Therefore, the elastic section modulus (S_x) was used to represent the dependent variable. The B_n coefficients are the regression coefficients associated with each independent variable. The X_n variables are independent variables, which include the unbraced length of member (L), the required axial strength (P_r), and the required flexure strength (M_r). The hypotheses of this study are defined as follows:

H0: no significant relationship exists between a dependent variable and independent variables.
H1: a significant relationship exists between a dependent variable and independent variables.

3. Results

3.1 The Database

In this study, the dataset employed to develop the multiple linear regression equation was 1,128 column steel design cases with square H-beam sections, which is allowable in Thailand. All data are divided into three groups based on the subject force of the member. These groups are as follows:

i. Dataset of the steel design member carrying the axial force.
ii. Dataset of the steel design member carrying the bending moment.
iii. Datasets of the steel design members carrying axial force and bending moment.

All datasets were rigorously verified according to design standards [28] to ensure strength and safety. The influencing parameters within the dataset were distributed as follows: The unbraced length (L) of the member varied from 4 to 10 meters, with 1 to 100 tons required axial strength magnitude (P_r) and 10 to 100 ton-meters required flexure strength (M_r). Moreover, the material grade of steel was SS400, which is commonly used in Thailand.

3.2 The Regression Model

Using the SPSS program kit in the case of multiple regression, the developed models and multiple linear regression analyses are shown below. The correlation between variables was checked based on the Pearson correlation coefficient in the first step. The results presented in Table 1 indicate no significant multicollinearity among the independent variables. This conclusion is based on the small values obtained from the Pearson correlation coefficient, which suggests that the variables do not have a strong linear relationship. Additionally, it is observed that the bending moment exhibits a strong correlation with the dependent variable, as indicated by a Pearson correlation coefficient of 0.982, which exceeds the threshold of 0.80. This finding aligns with the design theorem, which asserts that the elastic section modulus significantly influences the bending resistance of a member subjected to bending forces.

Table 1
The correlations between variables

		S_x	L	P_r	M_r
Pearson Correlation (r)	S_x	1.000	0.026	0.116**	0.982**
	L	0.026	1.000	0.000	0.000
	P_r	0.116**	0.000	1.000	-0.027
	M_r	0.982**	0.000	-0.027	1.000
Sig. (2-tailed)	S_x		0.380	0.000	0.000
	L	0.380		1.000	1.000
	P_r	0.000	1.000		0.371
	M_r	0.000	1.000	0.371	

**. Correlation is significant at the 0.01 level (2-tailed)

To assess the reliability of a multiple regression model, we employ the ANOVA analysis, which entails an examination of the variances summarized in Table 2. From Table 2, df represents the degree of freedom, and F represents the F-Statistic value obtained from the regression's mean square divided by the residual's mean square. The hypothesis H1 is accepted if the F value is greater than the F-Statistic table. Furthermore, the Sig.-value shown in Table 2 is 0.000, smaller than the significance level of 0.05.

Table 2
ANOVA analysis

Model		df	Sum of Squares (SS)	Mean Square (MS = SS/df)	F	Sig.
1	Regression	3	2764881758	921627253	24171	0.000*
	Residual	1124	42857666	38130		
	Total	1127	2807739424			

*. Significant at the 0.05 level

This result supports the alternative hypothesis H1, suggesting that there is a significant relationship between the dependent variable and the independent variables. The R-square value (0.985) of the multiple linear regression equation is shown in Table 3. This value is determined from the regression's sum of squares divided by the total sum of squares. The high R-square value indicates that the independent variables effectively estimate the dependent variable, indicating a reliable regression model fit.

Table 3
Model summary

Model	R	R Square	Adjusted R Square	Std. An Error of The Estimate	Durbin-Watson
1	0.992	0.985	0.985	195.268	2.349

Additionally, the normality of the residuals in a multiple linear regression analysis is an important assumption that should be assessed. If the residuals are normally distributed, the model's errors are random, and the regression analysis assumptions are satisfied. The histogram of the residual can be used to check whether the variance is normally distributed. A symmetric bell-shaped histogram evenly distributed around zero indicates that the normality assumption is likely valid. At the same time, the normal probability plot is the most widely used technique in residual analysis to verify this hypothesis. If the resulting plot is approximately linear, we proceed assuming that the error terms are normally distributed. From the SPSS program kit, the histogram and a normal probability plot of multiple linear regression are shown in Figure 5 to Figure 6. Observing that the histogram is symmetrical, and the normal probability plot is relatively linear suggests that the residuals of a multiple linear regression equation are normally distributed.

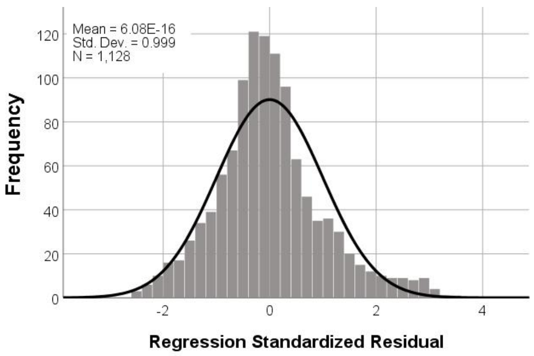

Fig. 5. The histogram of residual

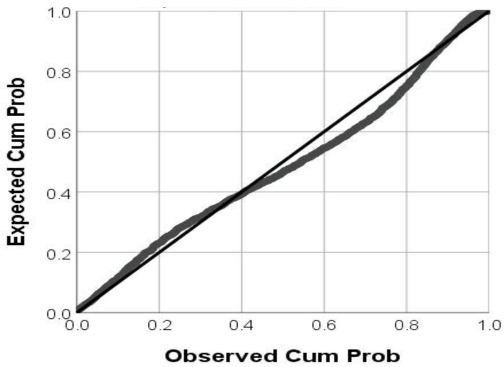

Fig. 6. The normal probability plot of residual

Overall, based on these results, it can be concluded that the independent variables have a significant relationship with the dependent variable, and the multiple linear regression equation provides a practical estimation of the dependent variable. The results of regression coefficients (B) are shown in Table 4. Moreover, Beta is the standardized coefficient; this value presents the influence of independent variables on the dependent variable. High values indicate that independent variables have a more significant effect on the dependent variable. Furthermore, t (t-test value) and Sig. (significance) describe which independent variables can be used in the equation. From the table, the values of Sig. for all variables are equal to 0.000, smaller than the significance level of 0.05. Therefore, all independent variables can be used in the multiple linear regression equation.

Table 4
The coefficient of independent variables

Model		Unstandardized Coefficient		Standardized Coefficient	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	-129.093	21.117		-6.113	0.000
	L	18.000	2.537	.026	7.094	0.000
	P_r	6.839	.177	.142	38.541	0.000
	M_r	47.193	.177	.986	267.348	0.000

From the results in Table 4, the multiple linear regression equation based on Eq. (1) can be rewritten as follows:

$$S_x = 18.000L + 6.839P_r + 47.193M_r - 129.093 \quad (2)$$

where S_x represents elastic section modulus (cm^3), L denotes unbraced length of member (m), P_r denotes required axial strength (tons), and M_r denotes required flexure strength (tons-m).

3.3 Steel Design by Regression Model

The first example demonstrates the effectiveness of Eq. (2). The beam-column element was considered to have a length of 4 meters and was subjected to axial and bending forces, as shown in Figure 7. The design axial force applied to the element was 20 tons, while the design bending force was 80 tons-m. By substituting these values into Eq. (2), the elastic section modulus was calculated as S_x = 18.000(4) + 6.839(20) + 47.193(80) – 129.093 = 3,855 cm^3.

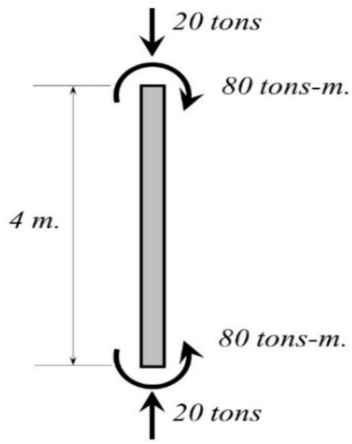

Fig. 7. Steel column subjected to axial and bending force

It should be noted that available steel cross-sections may not precisely match this calculated S_x value. In such situations, the designer may need to select a cross-section with a S_x value slightly larger or smaller than the estimated value based on the available steel options. The example steel cross-section in Table 5 was used in the specific example. The value obtained from Eq. (2) is between 3,540 cm^3 and 4,480 cm^3, closer to 3,540 cm^3. Therefore, the H400 cross-section with a weight of 197 kg/m (S_x = 3,540 cm^3) was chosen for this member. This cross-section was selected as the closest option that fulfilled the regression value requirements. Following the design standards in Thailand [28], the strength and safety of the selected cross-section have been checked.

Table 5
Example steel cross-sections

Section Index	Weight (kg/m)	Sectional Dimension (mm)				Area, A_g (cm^2)	Elastic Modulus, S_x (cm^3)
		d	b_f	t_w	t_f		
400 x 400	232	414	405	18	28	295.4	4,480
	197	400	408	21	21	250.7	3,540
	172	400	400	13	21	218.7	3,330
	147	394	398	11	18	186.8	2,850

Axial strength:
L_c/r_x = 400/16.8 = 23.8, F_{cr} = 2,389 ksc, then $\phi P_n = \phi F_{cr} A_g$ = 537 > 20 tons
Flexural strength:
L_p = 508 > 400, then $\phi M_n = \phi Z_x F_y$ = 86.4 > 80 tons-m
Beam-column strength capacity:
P_r/P_c = 0.037 < 0.2, then $(P_r/2P_c) + (M_r/M_c)$ = 0.93 < 1.00

By verifying the strength capacity of the member following the design criteria shown in Figure 4, it becomes evident that the selected cross-section can withstand the applied loads. The strength capacity value (0.94) shows that the cross-section chosen can be used with total efficiency. In other words, the selected cross-section is the most economical size. However, we can choose a cross-section size with a value of S_x greater than the value obtained from Eq. (2), which results in a stronger column but also a higher cost. The example shows that the S_x value obtained from Eq. (2) will be enough P_c, M_c for the P_r, M_r that the column must receive. Where P_c, M_c is referenced to the design standard in Figures 2 to 4 based on the member length. Also, the designer can use the P_r and M_r values calculated due to the *P-Δ* effect from structural analysis for long members.

From the example steel cross-sections in Table 5, it is apparent the cross-sectional areas of the same-size steel with the differences in thickness are very close. On the other hand, the difference between the S_x values is more noticeable. As discussed above, selecting S_x as the dependent variable is a better option. Furthermore, by the parameter values in the first example, the impact on the S_x value by another parameter can be observed when varying the values of each independent variable.

In the first case, the adjustment of the bending moment values is examined (P_r = 20 tons, L = 4 m.), as shown in Figure 8. The straight line is the value obtained from Eq. (2), and the circle mark is the cross-section selected from the steel table. Each number displayed is the strength capacity of the selected cross-section. It is evident from the figure that as the bending moment increases, the S_x value also increases significantly. This correlation aligns with the conclusions from the Pearson correlation analysis presented in Table 1.

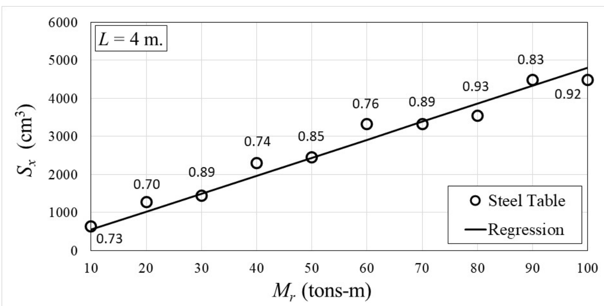

Fig. 8. influence of M_r on S_x

In the case of adjusting the axial force values (M_r = 80 tons-m, L = 4 m.), the results are shown in Figure 9, while Figure 10 illustrates the variation in S_x values with changes in the length of the element (P_r = 20 tons, M_r = 80 tons-m.). Both figures demonstrate that increasing the values of the respective variables has a minimal effect on the S_x value, as supported by the correlation analysis in Table 1. However, an increase in the independent variable will affect the strength capacity of the member. For example, from Figure 10, the strength capacity will increase at a length of 6-8 meters due to the reduced P_n and M_n values. Moreover, Figures 8 to 10 show that selecting a section from the steel tables based on the estimated S_x values makes it possible to choose a cross-section that closely matches the value obtained from the regression equation and is safe according to design standards. They were obtained from strength capacity value.

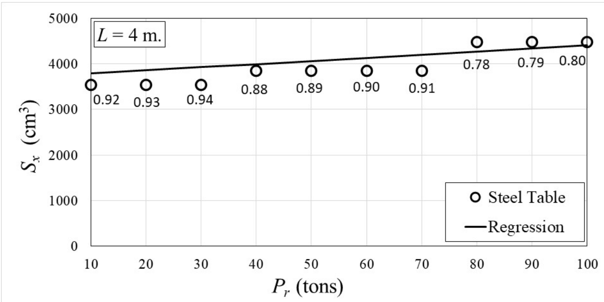

Fig. 9. influence of P_r on S_x

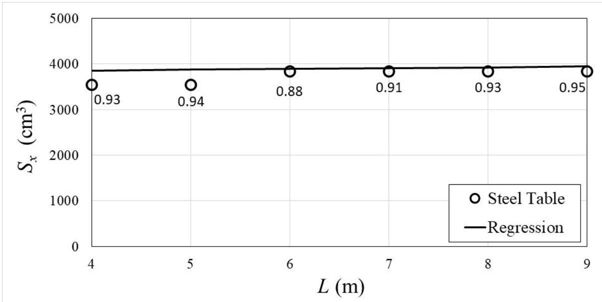

Fig. 10. influence of L on S_x

Moreover, Table 6 presents additional column steel designs examples for various cases to confirm the effectiveness of Eq. (2). These examples are shown to predict the elastic section modulus by Eq. (2) for different force conditions. Overall, by considering the predicted values and referring to the allowable steel section in Thailand, an appropriate cross-section can be confidently selected to satisfy the requirements and ensure the goodness of the strength capacity of the column under combined loading.

Table 6
Examples of column design under combined loading

No.	L (m)	P_r (tons)	M_r (tons-m)	Eq. (2) S_x (cm^3)	Section selected from steel table	strength capacity
1	4	30	20	1,092	H300-87kg (S_x =1,270 cm^3)	0.74
2	4	80	30	1,906	H350-115kg (S_x =1,940 cm^3)	0.85
3	5	40	40	2,122	H350-131kg (S_x =2,050 cm^3)	0.88
4	5	10	40	1,917	H350-115kg (S_x =1,940 cm^3)	0.90
5	6	70	20	1,401	H300-94kg (S_x =1,360 cm^3)	0.88
6	6	20	50	2,475	H350-156kg (S_x =2,450 cm^3)	0.87
7	7	50	80	4,114	H400-232kg (S_x =4,480 cm^3)	0.92
8	8	20	50	2,511	H400-147kg (S_x =2,850 cm^3)	0.79
9	9	30	40	2,126	H350-131kg (S_x =2,050 cm^3)	0.89
10	10	20	70	3,491	H400-172kg (S_x =3,330 cm^3)	0.92

The following example presents the practical application of Eq. (2) in designing beam and column steel members, considering bending forces only for beams and axial compression forces only for columns. The appropriate section is selected based on the estimated elastic section modulus (S_x) and assesses its safety compliance based on design standards, as detailed in Tables 7 and 8.

Table 7 shows the design examples for column steel members under compressive axial force. However, when designing members to resist axial forces, it is expected to determine the steel cross-sectional area (A_g) that can safely withstand them. The axial compressive strength depends on the cross-sectional area of the member, as shown in Figure 3. It is important to note that the value obtained from Eq. (2) is the elastic section modulus (S_x), not the cross-sectional area (A_g). Therefore, when performing safeness checks according to design criteria ($P_r < \phi P_n$), it is necessary to use the cross-sectional area (A_g) for the verification process. However, the designer can use the S_x value to select the section and obtain the A_g from the steel table. Table 7 shows that Eq. (2) can be applied to the column design, and the column section obtained from this equation can be used safely. Moreover, from No. 1 in Table 7, the S_x value from Eq. (2) and the S_x value from selected cross-section are very different because H100 - 17.2kg is the smallest square H-section that can be selected.

Table 7
Examples of column design

No.	L (m)	P_r (tons)	M_r (tons-m)	Eq. (2) S_x (cm^3)	Section selected from steel table	strength capacity
1	4	10	0	11	H100x17.2kg (S_x =76.5 cm^3)	0.58
2	5	30	0	166	H125x23.8kg (S_x =136 cm^3)	0.70
3	6	50	0	321	H175x40.2kg (S_x =330 cm^3)	0.62
4	7	40	0	270	H175x40.2kg (S_x =330 cm^3)	0.56

Similarly, Table 8 presents the design example for steel beam members. The value obtained from Eq. (2) can be used to select a cross-section with a S_x value close to the estimated value. The selected cross-section can be safely used when verified for strength capacity according to the design standards ($M_r < \phi M_n$).

Table 8
Examples of beam design

No.	L (m)	P_r (tons)	M_r (tons-m)	Eq. (2) S_x (cm^3)	Section selected from steel table	strength capacity
1	4	0	30	1,359	H300x94kg (S_x =1,360 cm^3)	0.90
2	5	0	50	2,321	H350x137kg (S_x =2,300 cm^3)	0.89
3	9	0	80	3,808	H400x200kg (S_x =3,840 cm^3)	0.91
4	10	0	90	4,298	H400x232kg (S_x =4,480 cm^3)	0.89

4. Conclusions

This study applied the multiple linear regression (MLR) theory to predict the size of beam-column members in steel design. Estimating the elastic section modulus (S_x) relied on three independent variables: length (L), design axial force (P_r), and design bending moment (M_r). Relationships between dependent and independent variables were examined using Pearson correlation and ANOVA. A predictive equation was formulated and expressed as Eq. (2) using multiple linear regression theorem. The precision and efficacy of the regression equation were evaluated by observing a higher R-square value and examining the distribution of residuals through a histogram and normal probability plot.

Additionally, two examples were executed to assess the performance of the predictive equation. The first example demonstrates the application of multiple linear regression equations in designing the beam-column member. The safety of the cross-section obtained from the predictive equations was checked with the design standards. The second example will be the application of the equations obtained in the research to the design of other types of members. The study results found that multiple linear regression equations in this study can be applied to estimate the size of steel beams and steel columns as well.

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